Probability Models for Rolling Irregular Dice in 2-Dimensions with Special Cases in 3-Dimensions

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Abstract

We have proven probability models for rolling 2-Dimensional Dice in an attempt to extend these results for 3-Dimensional objects, an approach which has largely been unconsidered. The problem motivating this research is the shaved dice problem. The new probabilities for any face on such a die is altered, but it is currently unknown to what degree. *Keywords:* Shaved Dice, Convex Polygons, Probability

Introduction

Instead of a solid object rolling in 3-Dimensions, consider rolling a convex polygon in a plane. A **convex polygon** is a polygon whose interior angles are less than 180° (see Figure 1, A and B). Whenever we say polygon, we mean a convex polygon. We imagine that there is a 2-Dimensional force which acts like gravity, pushing the polygon down until it finally rests on a side on the **floor** (the surface orthogonal to this downward force). Once the polygon touches the floor, it is in a **state**. A state is **final** if there is a line segment perpendicular to the floor that goes through the center of mass and the distance between the floor and the intersection of this line with the polygon's boundaries is zero (see Figure 1, C and D below).

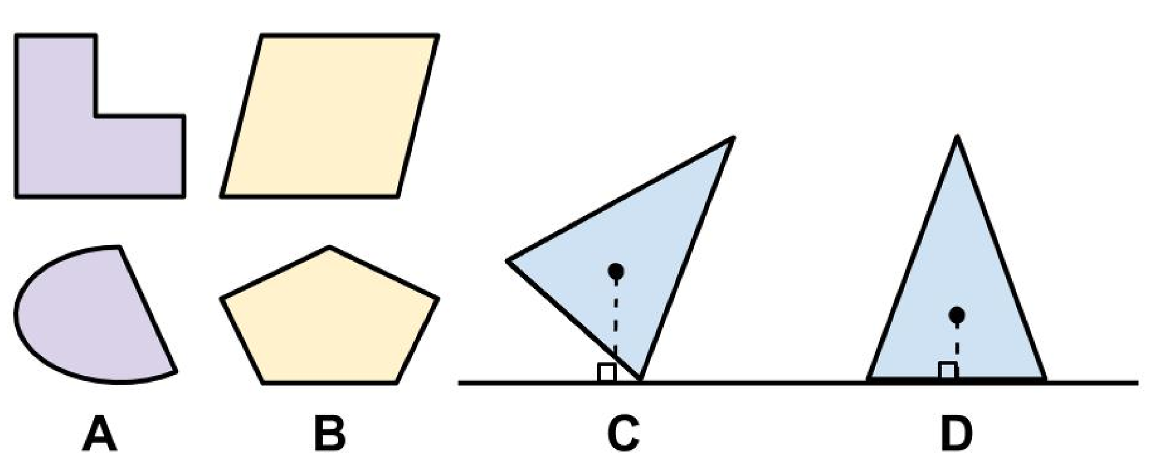

Figure 1: Between A and B, only the shapes in column B are convex polygons. The black dot in C and D marks the center of mass. C shows a polygon in a state, while D is a final state of C.

When a polygon lands, observe that it must first hit a vertex, then it will choose a side.

2-Dimensions

Theorem 1.1 The probability of a polygon initially landing on any side is zero. The probability of a vertex A being the vertex involved in the initial state is given by A's exterior angle divided by the sum of exterior angles. This gives the formula:

$$P(A) = \frac{180° - A_{\text{interior}}}{360°}.$$

Theorem 1.1 gives us the probabilities of any vertex hitting the floor, but once that happens, the polygon must then choose a side.

Theorem 1.2 The probability of a convex polygon rolling from a vertex V_{AB} to its adjacent sides A and B can be found using the following process:

1. Construct the line that goes between the center of mass and the vertex.
2. Construct a perpendicular line to this segment that goes through V_{AB}.
3. Take the smallest angle between A and this line and call it α. Do the same for B and call it β. If either of these angles are contained within the polygon, set that angle equal to zero.
4. $P(A|V_{AB}) = \frac{\alpha}{\alpha+\beta}$ and $P(B|V_{AB}) = \frac{\beta}{\alpha+\beta}$.

When choosing a side, if the center of mass is beyond a vertex, it will topple over, like Figure 2.

Theorem 1.3 Let A be a side of a polygon where the endpoints of A are V_1 and V_2 and let C be the center of mass. Then this state is final if and only if the angles $\angle CV_1V_2$ and $\angle CV_2V_1$ are both acute. Moreover, if one of these angles is obtuse, the state will transfer to the associated vertex state.

Using our three theorems, consider the figures below.

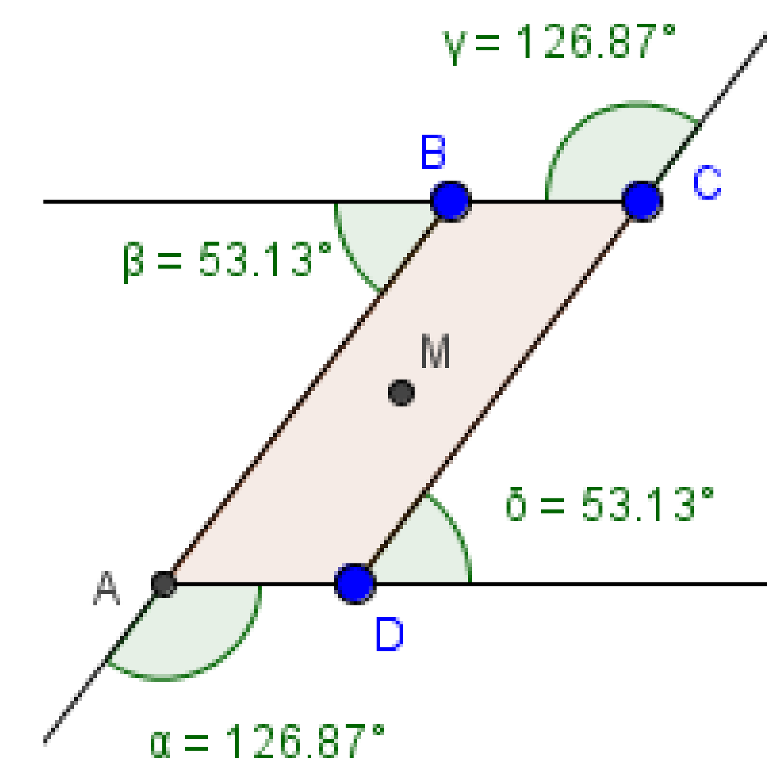

Figure 2: This is not a final state for the polygon. By Theorem 1.3, it will fall to $\overline{CD}$ since $\angle MDA$ is obtuse (M is the center of mass).

Theorem 1.3 will force Figure 2 to land on only $\overline{AB}$ and $\overline{CD}$. Using Theorems 1.1 and 1.2, we can calculate the probabilities for the sides of Figure 3. Since Figure 3 has similar angles, the probability for $\overline{AB}$ is the same as $\overline{BC}$. Thus, we only need to consider two vertices.

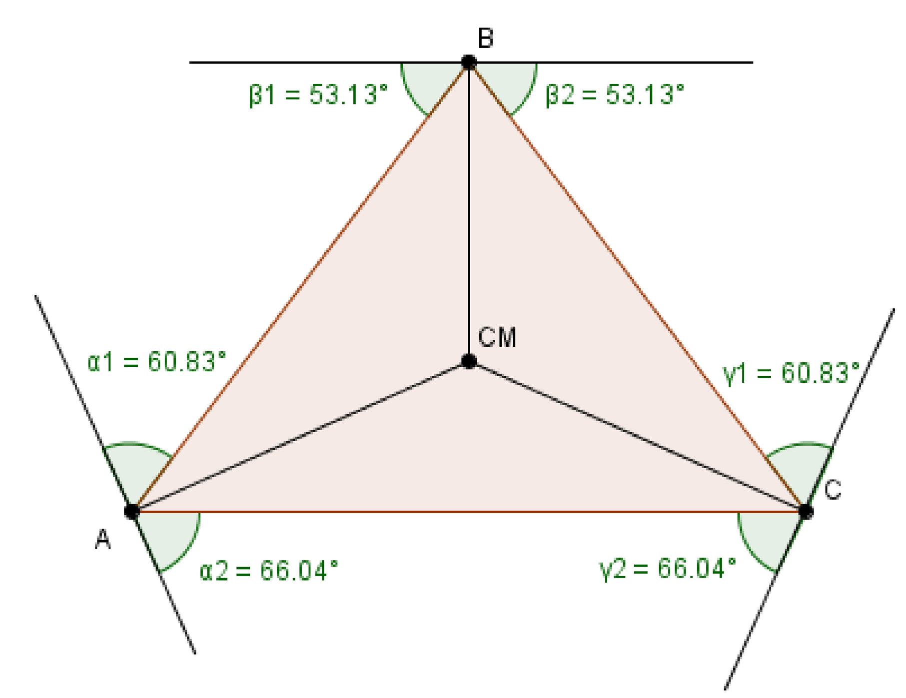

Figure 3: By Theorem 1.2, we construct lines perpendicular to the line segment made by the center of mass and the vertex.

If we continue to follow the steps in Theorem 1.2 for each side (see Figure 3), we obtain the following results:
$P(\overline{AC}) = 2(.3524)(.5205) = .3668$ and
$P(\overline{AB}) = P(\overline{BC}) \approx .3524(.4795) + .2952(.5) = .3166$.
Figure 4 is the results viewed as a chain.

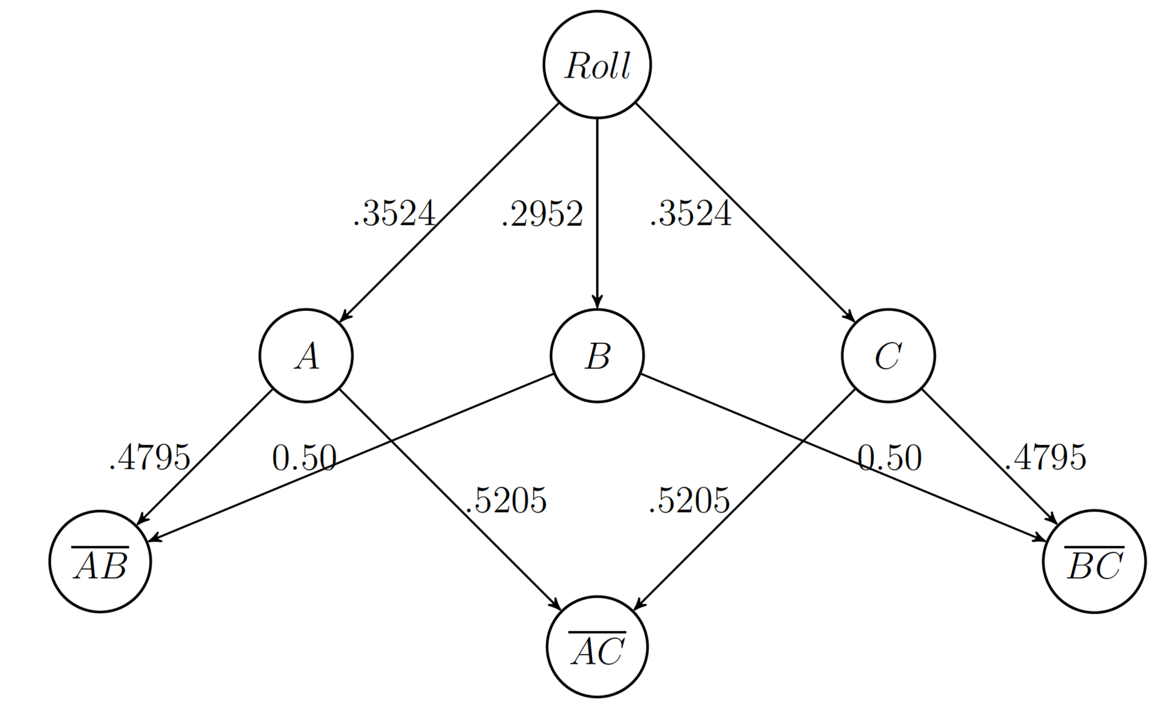

Figure 4: Each path from *Roll* to a side is a probability.

3D and Future Research

A 3D model can be built by layering similar but smaller polygons on each side, with the center of mass of each figure on the same axis. Figure 3 would thus form an object like Figure 5.

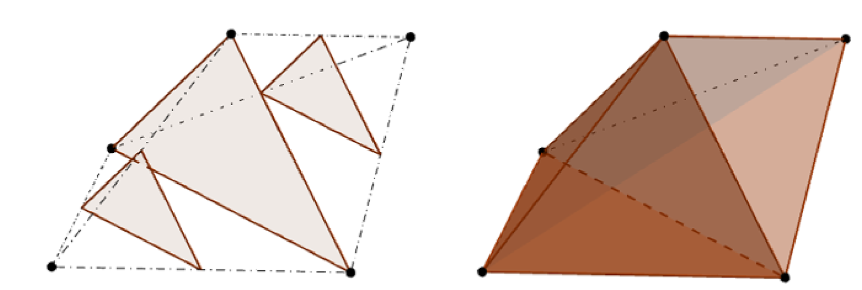

Figure 5: 3D representation of Figure 3

The probabilities for each surface of Figure 5 will be related to the polygon used to create it.

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